Course Title:	Advanced Placement (AP) PreCalculus
Grade Level(s):	10, 11, 12
Length of Course:	Two semesters or equivalent term
Credit:	10 units
Prerequisite:	Completion of Algebra II with a minimum of "C" grade, or instructor consent
Co-requisite:	Not applicable

Course Overview:

Advanced Placement (AP) PreCalculus prepares students for other college-level mathematics and science courses. Through regular practice, students build deep mastery of modeling and functions, and examine scenarios through multiple representations. The course framework delineates content and skills common to college PreCalculus courses that are foundational for careers in mathematics, physics, biology, health science, social science, and data science.

Schools Offering:	Del Valle High School Granada High School Livermore High School Vineyard High School
Meets University of California Entrance Requirements:	Seeking " c" approval
Board Approval:	*Pending Board Approval*
Course Materials:	*PreCalculus*, 7th edition; Demana, Franklin D., Bert K. Waits, Gregory D. Foley, and Daniel Kennedy; PreCalculus: Graphical, Numerical, Algebraic. 7th edition. Pearson Education Inc, copyright 2007 ISBN 013227650-X:

	Website - https://apcentral.collegeboard.org/courses/ap-precalculus/course
Supplemental Materials:	Will vary based on available websites and additional textbooks to cover the PreCalculus material

Advancement Placement (AP) PreCalculus

COURSE CONTENT:

AP Precalculus centers on functions modeling dynamic phenomena. This research-based exploration of functions is designed to better prepare students for college-level calculus and provide grounding for other mathematics and science courses. In this course, students study a broad spectrum of function types that are foundational for careers in mathematics, physics, biology, health science, business, social science, and data science. The course framework is organized into four commonly taught units of study that offer the following sequence for the course:

Unit 1: Polynomial and Rational Functions

In this unit, students develop understanding of two key function concepts while exploring polynomial and rational functions. The first concept is covariation, or how output values change in tandem with changing input values. The second concept is rates of change, including average rate of change, rate of change at a point, and changing rates of change. The central idea of a function as a rule for relating two simultaneously changing sets of values provides students with a vital tool that has many applications, in nature, human society, and business and industry. For example, the idea of crop yield increasing but at a decreasing rate or the efficacy of a medicine decreasing but at an increasing rate are important understandings that inform critical decisions.

Summary of Key Assignments and/or Activities
Students will explore rates of change in the following contexts and to answer the following questions, among others:

- How do we model the intensity of light from its source?
- How can I use data and graphs to figure out the best time to purchase event tickets?
- How can we adjust known projectile motion models to account for changes in conditions?

Sample activity: Students will be able to translate a given non-constant polynomial or rational function into a variety of analytical representations: constructing a graph, writing the expression as a product of linear factors (x-a) when possible, and verbally describing characteristics such as real zeroes, x-intercepts, asymptotes, and holes. Then students are able to check their graphs and analysis using technology.

Unit 2: Exponential and Logarithmic Functions

In this unit, students build an understanding of exponential and logarithmic functions. Exponential and logarithmic function models are widespread in the natural and social sciences. When an aspect of a phenomenon changes proportionally to the existing amount, exponential and logarithmic models are employed to harness the information. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of these two function types touches careers in business, medicine, chemistry, physics, education, and human geography, among others.

Summary of Key Assignments and/or Activities
Students will explore exponential and logarithmic functions in the following contexts and to answer the following questions, among others:
- How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end?
- How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see?
- If different functions can be used to model data, how do we pick which one is best?

Sample activity: students are given multiple tables of data, each of which is well-modeled by a linear, exponential, or quadratic function. In pairs or small groups, students calculate all three regressions, observe the graphs of the regressions, observe the corresponding residual plots, and draw conclusions about the relationships between the graphs and the residuals.

Unit 3: Trigonometric and Polar Functions

In this unit, students explore trigonometric functions and their relation to the angles and arcs of a circle. Since their output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena, such as: the highs and lows of a wave, the blood pressure produced by a heart, and the angle from the North Pole to the Sun year to year. Furthermore, periodicity is found in human inventions and social phenomena. For example, moving parts of an analog clock are modeled by a trigonometric function with respect to each other or with respect to time; traffic flow at an intersection over the course of a week demonstrates daily periodicity; and demand for a particular product over the course of a year falls into an annually repeating pattern. Polar functions, which are also explored in this unit, have deep ties to trigonometric functions as they are both based on the circle. Polar functions are defined on the polar coordinate system that uses the circular concepts of radii and angles to describe location instead of rectangular concepts of left-right and up-down, which students have worked with previously. Trigonometry serves as the bridge between the two systems.

Summary of Key Assignments and/or Activities
Students will explore trigonometric and polar functions in the following contexts and to answer the following questions, among others:
- How do we model aspects of circular and spinning objects without using complex equations from the x-y rectangular-based coordinate system?
- Since energy usage goes up and down through the year, how can I use trends in data to predict my monthly bills when I get my first apartment?
- How does right triangle trigonometry from geometry relate to trigonometric functions?

Sample activity: students are given analytical trigonometric functions paired with graphs on a single sheet of paper or on index cards. Some of the graphs are correct, and others are incorrect.

In pairs or small groups, students identify the function(s) for which the analytical and graphical representations are consistent and those for which the representations are inconsistent. For graphs that are in error, students construct the appropriate graph.

Unit 4: Functions Involving Parameters, Vectors, and Matrices

In this unit, students explore function types that expand their understanding of the function concept. Parametric functions have multiple dependent variables' values paired with a single input variable or parameter. Modeling scenarios with parametric functions allows students to explore change in terms of components. This component-based understanding is important not only in calculus but in all fields of the natural and social sciences where we seek to understand one aspect of a phenomenon independent of other confounding aspects. Another major function type in this unit involves matrices mapping a set of input vectors to output vectors. The capacity to map large quantities of vectors instantaneously is the basis for vector-based computer graphics. While students may see their favorite video game character trip and fall or seemingly move closer or farther, matrices implement a rotation on a set of vectors or a dilation on a set of vectors. The power of matrices to map vectors is not limited to graphics but to any system that can be expressed in terms of components of vectors such as electrical systems, network connections, and regional population distribution changes over time. Vectors and matrices are also powerful tools of data science as they can be used to model aspects of complex scientific and social science phenomena.

Summary of Key Assignments and/or Activities
Students will explore parametric functions, vectors and matrices in the following contexts and to answer the following questions, among others:

- How can we determine when the populations of species in an ecosystem will be relatively steady?
- How can we analyze the vertical and horizontal aspects of motion independently?
- How does high resolution computer-generated imaging achieve smooth and realistic motion on screen with so many pixels?

Sample activity: students are provided with four graphs: a graph of a parabola that opens up or opens down with the vertex at the origin, a circle centered at the origin, an ellipse not centered at the origin, and a hyperbola with center at the origin. In groups, students develop a reason for why each of the four graphs does not belong to the set of four. (For example: The parabola is the only one that can be explicitly defined with y as a function of x; the circle is the only one with infinitely many lines of symmetry; the ellipse does not belong because it is not symmetric to the y-axis; the hyperbola does not belong because it has two disconnected pieces.) This type of activity can be repeated with four different graphs of a parabola, a circle, an ellipse, and a hyperbola, where students come up with the graphs.

PreCalculus Chapter of the Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve

Standards for Mathematical Practice

Students will be able to do mathematics and experience mathematics as a coherent, useful, and logical subject.

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others. Students build proofs by induction and proofs by contradiction.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

The Complex Number System

N-CN 3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N-CN 4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N-CN 5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-\ 1\ +\sqrt{3}i)^3 = 8$ because $(-\ 1\ +\sqrt{3}i)$ has modulus 2 and argument 120°.
N-CN 6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Vector and Matrix Quantities

N-VM 1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v, |v|, ||v||)**
N-VM 2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N-VM 3. Solve problems involving velocity and other quantities that can be represented by vectors.
N-VM 4. Add and subtract vectors.
- 4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- 4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- 4.c Understand vector subtraction **v-w** as **v+(-w)** where **-w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM 5. Multiply a vector by a scalar
- 5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise.

5.b Compute the magnitude of a scalar multiple **cv** using ||**cv**|| = |c|**v**. Compute the direction of **cv** knowing that when |c|**v** ≠ **0**, the direction of **cv** is either along **v** (for c > 0) or against **v** (for c < 0).

N-VM 6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM 7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

N-VM 8. Add, subtract, and multiply matrices of appropriate dimensions.

N-VM 9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM 10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is non-zero if and only if the matrix has a multiplicative inverse.

N-VM 11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

N-VM 12. Work with 2x2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Seeing Structure in Expressions

A-SSE 1. Interpret expressions that represent a quantity in terms of its context.

1.a Interpret parts of an expression, such as terms, factors, and coefficients.

1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.

A-SSE 2. Use the structure of an expression to identify ways to rewrite it.

Arithmetic with Polynomials and Rational Expressions

A-APR 6. Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for more complicated examples, a computer algebra system.

A-APR 7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a non-zero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

A-CED 1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A-CED 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s Law V = IR to highlight resistance R.

Reasoning with Equations and Inequalities

A-REI 8. Represent a system of linear equations as a single matrix equation in a vector variable.
A-REI 9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimensions 3 x 3 or greater).

Interpreting Functions

F-IF 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior, and periodicity.
F-IF 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   7.d Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   7.e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F-IF 10. Demonstrate an understanding of functions and equations defined parametrically and graph them.
F-IF11. Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems.

Building Functions

F-BF 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, k$f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F-BF 4. Find inverse functions
   4.b Verify by composition that one function is the inverse of another.
   4.c Read values of an inverse function from a graph or table, given that the function has an inverse.
   4.d Produce an invertible function from a non-invertible function by restricting the domain.

Trigonometric Functions

F-TF 4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F-TF 6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F-TF 7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F-TF 9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
F-TF 10. Prove the half angle and double angle identities for sine and cosine and use them to solve problems.

Similarity, Right Triangles, and Trigonometry

G-SRT 9. Derive the formula $A = \frac{1}{2}ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G-SRT 10. Prove the Laws of Sines and Cosines and use them to solve problems.
G-SRT 11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations

G-GPE 3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of the distances from the foci is constant.
G-GPE 3.1 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method for completing the square to get the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola and graph the equation.

Instructional Methods and/or Strategies

- Lecture and large group discussion
- Individual work using concrete problem-solving methodology
- Group work using whiteboards and other modes for students to discuss difficult mathematical concepts and devise solutions
- Internet Tutorials and lectures
- Peer-Tutoring
- Student research-based presentations
- Activity and laboratory assignments to build concrete multivariable Calculus understanding

Assessment Methods and/or Tools

- Pretests and/or Quizzes on Concepts
- Post Unit Tests
- Final end-of-course examination
- Portions of assessments require the use of a graphing calculator and other portions do not allow a calculator
- Multiple Choice and Free Response Questions on exams
- Exit Tickets - students are expected to be able to solve a problem from the content of the course that day upon leaving the classroom
- Assessment interviews - students are expected to be able to explain a concept or application upon questioning - “one-on-one” with instructor

Honors Courses

Every AP course is designed in consultation with college faculty and experienced high school teachers. In an ongoing effort to maintain alignment with best practices in college-level learning, AP courses and exams emphasize research-based curricula aligned with higher education expectations. College faculty and experienced high school teachers guide the development of the AP course framework, which defines what students must know and be able to do to earn a qualifying score on the AP Exam, thus conferring college credit or placement.

As part of the course development process for *AP PreCalculus*, the AP Program gathered course research through examination of college syllabi, analysis of textbooks and pedagogical research, and content advisory sessions with college faculty. Then, an advisory board and writing team collaborated on the course framework based on these research inputs.